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Multiparticle correlations from momentum conservation

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Abstract. Using a generating-function formalism, we compute the contribution of momentum conservation to multiparticle correlations between the emitted particles in high-energy collisions. In particular, we derive a compact expression of the genuine M-particle correlation, for arbitrary M.

The main purpose of this paper is to deal with a rather general problem: consider N random variables (in a D-dimensional space) $\mathbf{p}_1, \ldots, \mathbf{p}_N$, which are independent except for a constraint $\mathbf{p}_1 + \ldots + \mathbf{p}_N = \mathbf{0}$. What is the multiple correlation between M variables among the $\mathbf{p}_j$ (with $M < N$) induced by this constraint, and in particular, what is the *cumulant* of the M-variable correlation?

One may for example think of the correlation between M monomers of a finite-size-N ring polymer. Another instance of the problem under study, from which we borrow the terminology in this paper, is the constraint arising from global momentum conservation in high-energy collisions: in the center-of-mass frame, the sum of the momenta of the N emitted particles vanishes, and this induces correlations between M arbitrary particles.

An accurate knowledge of these unavoidable multiparticle correlations due to global momentum conservation is important. They represent an ever-present background effect to other sources of correlations between the outgoing particles, which are a primary target of investigation, as they provide information on the physics involved in the collisions [1]: either short-range correlations, as quantum (Bose–Einstein or Fermi–Dirac) effects [2–4], correlations between decay products, (mini)jets, etc.; or inter-particle correlations arising from a collective motion ("flow") in the context of heavy-ion collisions [5]. For instance, it has been shown that the analysis of collective flow can be biased by the two-particle correlations due to global momentum conservation [6]

A common property of short-range correlations is their scaling. If N denotes the total number of emitted particles, the connected part of the M-particle correlation scales as $1/N^{M-1}$. This scaling follows from simple combinatorics: $1/N^{M-1}$ is the probability that M arbitrary particles have been together in a small phase-space region.

Global momentum conservation is certainly not a short-range correlation, since it affects *all* particles in the collision. One may therefore wonder how the resulting correlation scales with N for a given M. The two-particle case has already been calculated [6,7]. In the following, we extend the calculation to M-particle correlations for arbitrary M, using a generating function of the correlations and a saddle-point approximation. In particular, we shall derive a compact expression for the correlation at any order M, to leading order in $1/N$, and show that it follows the same behavior as short-range correlations.

We first introduce, in Sect. 1, a generating function of the multiparticle correlations. This generating function then allows us to derive the expression of the genuine correlation (cumulant) between an arbitrary number of particles to leading order in $1/N$ (Sect. 2), and in particular the scaling of the cumulants. As an illustration of this result, we then compute explicitly the two- and three-particle correlations (Sect. 3). The results are commented on in Sect. 4.

1 Multiparticle cumulants

Consider a collision in which a total of N particles are emitted (throughout this paper, N is assumed to be large). Let $\mathbf{p}_1, \ldots, \mathbf{p}_N$ be their D-dimensional momenta, where the space dimension D is kept arbitrary for the sake of generality. For a given $M < N$, we denote by $f(\mathbf{p}_{j_1}, \ldots, \mathbf{p}_{j_M})$ the M-particle probability distribution of $\mathbf{p}_{j_1}, \ldots, \mathbf{p}_{j_M}$.

To derive multiparticle correlations of arbitrary order in a systematic way, we introduce a generating function of the distributions, which we define as

$$\begin{aligned} G(x_1, \ldots, x_N) \equiv 1 + x_1 f(\mathbf{p}_1) + x_2 f(\mathbf{p}_2) + \cdots \\ + x_1 x_2 f(\mathbf{p}_1, \mathbf{p}_2) + \cdots, \end{aligned} \tag{1}$$

and so on for every order M. Thus, the coefficient of the term $x_{j_1} x_{j_2} \ldots x_{j_M}$ is the M-particle probability distribution $f(\mathbf{p}_{j_1}, \mathbf{p}_{j_2}, \ldots, \mathbf{p}_{j_M})$.

We shall mostly be interested in the *connected part* of the M-particle distribution, that is, the term which cannot be expressed as a product of correlations between less

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than M particles. For instance, the two-particle distribution can be decomposed into two parts:

$$f(\mathbf{p}_1,\mathbf{p}_2) = f(\mathbf{p}_1)\, f(\mathbf{p}_2) + f_c(\mathbf{p}_1,\mathbf{p}_2),$$

where the first term on the right-hand side (RHS) is the mere product of the one-particle probability distributions, while the second one, the "connected part," reflects correlations between $\mathbf{p}_1$ and $\mathbf{p}_2$. In particular, $f_c(\mathbf{p}_1,\mathbf{p}_2)$ vanishes if $\mathbf{p}_1$ and $\mathbf{p}_2$ are uncorrelated. This genuine correlation, which is also called the *cumulant* of the distribution in probability theory, is obtained by taking the logarithm of the generating function of distributions [8]. The cumulant $f_c(\mathbf{p}_{j_1},\ldots,\mathbf{p}_{j_M})$ is the coefficient of $x_{j_1}\ldots x_{j_M}$ in the expansion of $\ln G(x_1,\ldots,x_N)$:

$$\ln G(x_1,\ldots,x_N) \equiv x_1 f_c(\mathbf{p}_1) + x_2 f_c(\mathbf{p}_2) + \cdots + x_1x_2 f_c(\mathbf{p}_1,\mathbf{p}_2) + \cdots . \tag{2}$$

If the system splits into independent subsystems or, more generally, if correlations in the system are short-range, the probability distributions of each subsystem add: $f(\{\mathbf{p}_j\}) = \sum_A (N_A/N) f_A(\{\mathbf{p}_j\})$, where the sum runs over subsystems with sizes N_A. It follows that G can be factorized into the product of functions for each subsystem: $G(\{x_j\}) = \prod_A g_A(\{N_A x_j/N\})$. In the limit where each individual particle is a subsystem, $G(\{x_j\}) = [g(\{x_j/N\})]^N$. Therefore, $\ln G$, and the cumulants, are the sums of the corresponding quantities for the subsystems. As a result, inspecting the term in $x_{j_1}\ldots x_{j_M}$ shows that, while the M-particle probability distribution $f(\mathbf{p}_{j_1},\ldots,\mathbf{p}_{j_M})$ is independent of N, the cumulant $f_c(\mathbf{p}_{j_1},\ldots,\mathbf{p}_{j_M})$ scales like $1/N^{M-1}$. In Sect. 2, we shall show that the cumulants from the correlations due to global momentum conservation follow the same scaling to leading order in $1/N$. More precisely, we shall demonstrate that the generating function of distributions reads

$$G(x_1,\ldots,x_N) \propto \mathrm{e}^{Ng(\{x_j/N\})}\left(1+\sum \frac{\{x_j\}^l}{N^q}\right),$$

where the sum runs over terms with $q \geq l$.

2 Cumulants in the large-N limit

In this section, we derive a compact expression of the cumulants of multiparticle correlations due to global momentum conservation to leading order in $1/N$. In particular, we show that the M-particle cumulants scale like $1/N^{M-1}$.

Let us assume for simplicity that the only source of correlation between the particles is global momentum conservation, $\mathbf{p}_1+\cdots+\mathbf{p}_N = \mathbf{0}$. Under this assumption, the M-particle distribution of $\mathbf{p}_1, \ldots, \mathbf{p}_M$, with $M<N$, is defined as

$$\begin{aligned} &f(\mathbf{p}_1,\ldots,\mathbf{p}_M) \\ &\equiv \left\{ \left(\left(\prod_{j=1}^{M} F(\mathbf{p}_j)\right) \int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N) \right.\right. \\ &\quad \left. \times \prod_{j=M+1}^{N} \left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]/\mathcal{N}_D^{N-M} \right) \Bigg/ \\ &\quad \left.\left(\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N) \prod_{j=1}^{N}\left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]/\mathcal{N}_D^N \right)\right\}, \end{aligned} \tag{3}$$

where $F(\mathbf{p})$ is the one-particle momentum distribution unrenormalized for the momentum conservation constraint, and $\mathcal{N}_D \equiv \int F(\mathbf{p})\,\mathrm{d}^D\mathbf{p}$ is a normalization constant. In the following, we denote by $\langle\ldots\rangle$ an $F(\mathbf{p})$-weighted average, that is, $\langle g(\mathbf{p})\rangle \equiv \int g(\mathbf{p})F(\mathbf{p})\,\mathrm{d}^D\mathbf{p}/\mathcal{N}_D$, for any function of momentum $g(\mathbf{p})$.

The denominator in (3) is a constant, which we denote by $1/\mathcal{C}_D$, independent of M. The actual value will not influence the forthcoming discussion.

Consider next the numerator of (3). Introducing a Fourier representation of the Dirac distribution, it reads

$$\begin{aligned} &\int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\left(\prod_{j=1}^{M} F(\mathbf{p}_j)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}\right)\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle^{N-M} \\ &= \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle^{N}\left(\prod_{j=1}^{M} F(\mathbf{p}_j)\frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle}\right). \end{aligned} \tag{4}$$

Inserting in (1) the expression of the M-particle distribution, (3), with the numerator replaced by the RHS of (4), we obtain

$$\begin{aligned} &G(x_1,\ldots,x_N) \\ &= \mathcal{C}_D \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle^{N}\prod_{j=1}^{N}\left(1+x_jF(\mathbf{p}_j)\frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle}\right) \\ &\simeq \mathcal{C}_D \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle^{N}\exp\left(\sum_{j=1}^{N} x_jF(\mathbf{p}_j)\frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle}\right). \end{aligned} \tag{5}$$

In passing from the first line to the second one, we have used the fact that we shall only consider the coefficient of $x_{j_1}\ldots x_{j_M}$ where the M indices $j_1, \ldots, j_M$ are all different [see (2)]. One easily checks that the only difference between the two forms of the generating function in (5) comes from terms in which at least one x_j is raised to some power $m \geq 2$ (corresponding to the "autocorrelation" of particle j with itself). Therefore, as far as we are concerned, (5) really is an *identity*, not an approximation.

Please note that in (5) the variable x_j is always multiplied by a factor $F(\mathbf{p}_j)$. We may thus rescale x_j by this factor, and drop it in the following to simplify expressions. This is quite satisfactory, since it means that the measurable multiparticle correlations $f(\mathbf{p}_{j_1},\ldots,\mathbf{p}_{j_M})$ or $f_c(\mathbf{p}_{j_1},\ldots,\mathbf{p}_{j_M})$ will not depend on the non-measurable distribution $F(\mathbf{p})$ – this justifies our previously calling it "unrenormalized".

To evaluate the integral in (5), we rely on the fact that N is large, and use a saddle-point approximation. To simplify the discussion, we introduce the notation

$$\mathcal{F}(\mathbf{k}) \equiv \ln\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle + \sum_{j=1}^{N} \frac{x_j}{N} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle}. \tag{6}$$

Thus, the integrand in (5) is $\mathrm{e}^{N\mathcal{F}(\mathbf{k})}$. We call $\mathbf{k}_0$ the position of the saddle point. Please note that $\mathcal{F}$ and its successive derivatives, which we shall denote by $\mathcal{F}'$, $\mathcal{F}''$, $\mathcal{F}^{(3)}$, ..., depend on x only through x/N, where x stands for any of the x_j. Therefore, $\mathbf{k}_0$, which is of course the solution of $\mathcal{F}'(\mathbf{k}) = 0$, also depends on x/N only. One should pay attention to the fact that the saddle point $\mathbf{k}_0$ is not merely the origin $\mathbf{k} = \mathbf{0}$.

We shall now demonstrate that to leading order in $1/N$, all multiparticle cumulants are determined by the saddle-point value $N\mathcal{F}(\mathbf{k}_0)$. More precisely, we show that the M-particle cumulant is of order $1/N^{M-1}$, and that corrections to the saddle-point calculation only yield subleading terms, suppressed by (positive) powers of $1/N$.

Using a Taylor expansion of $\mathcal{F}(\mathbf{k})$ around the saddle point, the generating function (5) reads

$$\begin{aligned}
&G(x_1,\ldots,x_N)\\
&= \mathcal{C}_D\, \mathrm{e}^{N\mathcal{F}(\mathbf{k}_0)} \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\, \mathrm{e}^{N\mathcal{F}''(\mathbf{k}_0)(\mathbf{k}-\mathbf{k}_0)^2/2}\\
&\qquad \times \exp\left[N \sum_{m=3}^{+\infty} \frac{\mathcal{F}^{(m)}(\mathbf{k}_0)}{m!}(\mathbf{k}-\mathbf{k}_0)^m\right]\\
&= \mathcal{C}_D\, \mathrm{e}^{N\mathcal{F}(\mathbf{k}_0)} \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\, \mathrm{e}^{N\mathcal{F}''(\mathbf{k}_0)(\mathbf{k}-\mathbf{k}_0)^2/2}\\
&\qquad \times \left(1 + \sum_{n=1}^{+\infty} \frac{1}{n!}\left[N \sum_{m=3}^{+\infty} \frac{\mathcal{F}^{(m)}(\mathbf{k}_0)}{m!}(\mathbf{k}-\mathbf{k}_0)^m\right]^n\right).
\end{aligned} \tag{7}$$

As recalled in Sect. 1, the cumulants are given by the logarithm of the generating function. Now, the logarithm of (7) will split into three parts. First, there is $\ln\mathcal{C}_D$, which does not depend on x, and thus does not influence the values of the multiparticle cumulants.

Next, there is $N\mathcal{F}(\mathbf{k}_0)$. As noted above, both $\mathcal{F}$ and $\mathbf{k}_0$ only involve powers of x/N. Thus, the coefficient of $x_{j_1}\ldots x_{j_M}$ in $\mathcal{F}(\mathbf{k}_0)$ contains a factor $1/N^M$. Multiplying by the overall factor of N, we find that the contribution of $N\mathcal{F}(\mathbf{k}_0)$ to the cumulant $f_c(x_{j_1},\ldots,x_{j_M})$ scales as $1/N^{M-1}$.

Finally, the cumulants involve the contribution of the logarithm of the integral in (7). We shall use the fact that the integrand is the sum of a Gaussian function and of combinations of its moments. After integration, this yields a common factor, $1/[2\pi N\mathcal{F}''(\mathbf{k}_0)]^{D/2}$, which multiplies a sum Σ. Let us show that Σ only involves terms in x^l/N^q with $q \geq l$. Noting that the values which contribute to the Gaussian integral are of order $(\mathbf{k}-\mathbf{k}_0)^2 \approx 1/N$, one sees that each term $N\mathcal{F}^{(m)}(\mathbf{k}_0)(\mathbf{k}-\mathbf{k}_0)^m$ is of order $N^{1-m/2}$, multiplied by a function of x/N. Since the sum runs over $m \geq 3$, $N^{1-m/2} \leq N^{-1/2}$; therefore, a power x^l goes with at most a factor $1/N^{l+1/2}$. Even after raising the sum to the power n, and taking the logarithm, this will always remain of the form x^l/N^q with $q > l$. Finally, the logarithm of the overall factor $1/[2\pi N\mathcal{F}''(\mathbf{k}_0)]^{D/2}$ involves x only through the form of powers of x/N in $\ln\mathcal{F}''(\mathbf{k}_0)$. All in all, the contribution to the M-particle cumulant of the integral is therefore at most of order $1/N^M$, subleading with respect to the contribution of $N\mathcal{F}(\mathbf{k}_0)$.

Therefore, we have shown that the leading contribution to the cumulants of multiparticle correlations due to momentum conservation is the saddle-point value: $\ln G(x_1,\ldots,x_N) = N\mathcal{F}(\mathbf{k}_0)$. This means first that the mere knowledge of $\mathbf{k}_0$ gives access to all cumulants at once, at least to leading order. One easily checks that $\mathbf{k}_0$ is pure imaginary, i.e., $\mathrm{i}\mathbf{k}_0$ has real-valued components. From (6), it follows that $\mathcal{F}(\mathbf{k}_0)$ is real-valued as well, and so are the cumulants, as they should be. We shall illustrate these points by computing explicitly the two- and three-particle cumulants in the following section. Our result also means that genuine M-particle correlations arising from momentum conservation, which is a long-range effect, scale in the same way as correlations from short-range sources, namely as $\mathcal{O}(1/N^{M-1})$. This was certainly not obvious a priori.

3 Two- and three-particle cumulants

As an example of the order of magnitude derived in the previous section, let us compute the two- and three-particle cumulants. According to the discussion in Sect. 1, they are given by the terms in x^2 and x^3 in the generating function of cumulants $\ln G(x_1,\ldots,x_N)$, that is, following Sect. 2, the corresponding terms in $N\mathcal{F}(\mathbf{k}_0)$. We shall for simplicity perform calculations assuming that $F(\mathbf{p})$ only depends on the modulus $|\mathbf{p}|$, not on the azimuthal angle of $\mathbf{p}$. As a consequence, the average momentum $\langle\mathbf{p}\rangle$ vanishes. Departures from this assumption are discussed at the end of this section.

Now a straightforward calculation shows that the saddle point $\mathbf{k}_0$ is given by

$$\left(\sum_{j=1}^{N} \frac{x_j}{N} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}}\rangle} - 1\right) \langle \mathbf{p}\mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}}\rangle = \sum_{j=1}^{N} \frac{x_j}{N}\mathbf{p}_j\, \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}_j}. \tag{8}$$

As mentioned above, $\mathbf{k}_0$ is a function of x/N. Equation (8) can be solved order by order in x/N, using the fact that one term (on the left-hand side) is of order 0 in x/N while the other two are linear in x/N. Inspecting (6), and using the fact that the first term in the RHS is even in $\mathbf{k}$, we see that the calculation of $\mathcal{F}(\mathbf{k}_0)$ to order x^3 requires our knowing $\mathbf{k}_0$ to order x^2 (while the calculation to order x^2 only requires $\mathbf{k}_0$ to order x).

We can then compute $\mathbf{k}_0$, expanding (8) in powers of $\mathbf{k}_0$, which gives

$$\mathrm{i}\mathbf{k}_0 = -\left[1_D - \left(X_0 1_D - \frac{D}{\langle\mathbf{p}^2\rangle}X_2\right)\right]^{-1} \frac{D}{\langle\mathbf{p}^2\rangle}\mathbf{X}_1, \tag{9}$$

where 1_D denotes the unit $D\times D$ matrix, and we have introduced

$$X_0 \equiv \sum_{j=1}^{N} \frac{x_j}{N}, \quad \mathbf{X}_1 \equiv \sum_{j=1}^{N} \frac{x_j}{N}\mathbf{p}_j, \quad X_2 \equiv \sum_{j=1}^{N} \frac{x_j}{N}\mathbf{p}_j\otimes\mathbf{p}_j.$$

Note that $\mathrm{i}\mathbf{k}_0$ is real-valued, as expected. Reporting its value in (6), we obtain

$$\mathcal{F}(\mathbf{k}_0) = X_0 - \frac{D}{2\langle\mathbf{p}^2\rangle}(\mathbf{X}_1)^2 - \frac{D}{2\langle\mathbf{p}^2\rangle}\mathbf{X}_1\cdot\left(X_0 1_D - \frac{D}{\langle\mathbf{p}^2\rangle}X_2\right)\cdot\mathbf{X}_1.$$

Multiplying this result by N, we finally obtain, to leading order in $1/N$

$$\ln G(x_1,\ldots,x_N) = \sum_{j=1}^{N} x_j - \frac{D}{2N\langle\mathbf{p}^2\rangle}\sum_{j,k} x_j x_k(\mathbf{p}_j\cdot\mathbf{p}_k) - \frac{D}{2N^2\langle\mathbf{p}^2\rangle}\sum_{j,k,l} x_j x_k x_l\left[\mathbf{p}_j\cdot\mathbf{p}_l - \frac{D}{\langle\mathbf{p}^2\rangle}(\mathbf{p}_j\cdot\mathbf{p}_k)(\mathbf{p}_k\cdot\mathbf{p}_l)\right] + \mathcal{O}(x^4). \tag{10}$$

Hence the coefficients of x_1x_2 and $x_1x_2x_3$:

$$f_c(\mathbf{p}_1,\mathbf{p}_2) = -\frac{D\,\mathbf{p}_1\cdot\mathbf{p}_2}{N\langle\mathbf{p}^2\rangle}, \tag{11}$$

$$f_c(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = -\frac{D}{N^2\langle\mathbf{p}^2\rangle}(\mathbf{p}_1\cdot\mathbf{p}_2+\mathbf{p}_1\cdot\mathbf{p}_3+\mathbf{p}_2\cdot\mathbf{p}_3) + \frac{D^2}{N^2\langle\mathbf{p}^2\rangle^2}\Big[(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_3)+(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_2\cdot\mathbf{p}_3) + (\mathbf{p}_1\cdot\mathbf{p}_3)(\mathbf{p}_2\cdot\mathbf{p}_3)\Big]. \tag{12}$$

We recover, in the case $D = 2$, the expression of the two-particle correlation due to *transverse* momentum conservation already derived in [6,7]: the correlation is back-to-back, and stronger between high-momenta particles.

Finally, let us comment on the assumption that $F(\mathbf{p})$ only depends on the modulus of $\mathbf{p}$, not on its azimuthal angle.

Our expansion of (8) relies on both $\langle\mathbf{p}\rangle = \mathbf{0}$ and on the identity $\langle(\mathbf{k}\cdot\mathbf{p})^2\rangle = \mathbf{k}^2\langle\mathbf{p}^2\rangle/D$, which are no longer valid if F is non-isotropic. In the general case, $\mathbf{k}^2\langle\mathbf{p}^2\rangle/D$ will be replaced by another quadratic form in $\mathbf{k}$, namely $\mathbf{k}\cdot\langle\mathbf{p}'\otimes\mathbf{p}'\rangle\cdot\mathbf{k}$, where $\mathbf{p}'\equiv\mathbf{p}-\langle\mathbf{p}\rangle$. To perform explicit calculations, it becomes necessary to introduce the principal-axes frame, in which $\mathbf{p}'\otimes\mathbf{p}'$ is diagonal. For instance, the two-particle cumulant, (11), will read

$$f_c(\mathbf{p}_1,\mathbf{p}_2) = -\sum_{i=1}^{D}\frac{(\mathbf{p}_1)_i(\mathbf{p}_2)_i}{N\langle(\mathbf{p})_i^2\rangle}, \tag{13}$$

where the sum runs over the coordinates along the principal axes. The distinction between the various directions is for example relevant in a high-energy collision: while the *transverse* momentum distribution can be isotropic (unless there is some anisotropy, as, e.g., a correlation to the impact parameter direction), the isotropy will not extend to the beam direction. In that case, one may want to use (11), with $D = 2$, when studying two-particle correlations in the transverse plane, but turn to (13) if interested in three-dimensional correlations.

4 Discussion

We have shown how it is possible to calculate the multiparticle correlations arising from momentum conservation between any number of particles using a generating-function formalism:

$$\ln G(x_1,\ldots,x_N) = N\mathcal{F}(\mathbf{k}_0), \tag{14a}$$

where $\mathcal{F}(\mathbf{k})$ is given by (6) and $\mathbf{k}_0$ by

$$\mathcal{F}'(\mathbf{k}_0) = 0. \tag{14b}$$

In particular, we have seen that the M-particle cumulant scales as $1/N^{M-1}$, where N is the total number of emitted particles. This means that the correlation scales in the same way as correlations arising from short-range final interactions or from resonance decays, although the underlying reason is less obvious. In the latter cases, the scaling is a simple consequence of combinatorics: the M particles are required either to be altogether in a small phase-space region, or to originate from a single decay, hence the factor $1/N^{M-1}$. However, in the case of global momentum conservation, the same arguments do not apply a priori.

The scaling of the genuine M-particle correlation due to momentum conservation has several consequences. First, a good feature: in the context of collective flow in heavy-ion collisions [5], a new method of analysis has been proposed [9], which relies on the idea that a cumulant expansion allows one to separate genuine collective phenomena from trivial short-range correlations, while they interfere in the data. Since correlations from momentum conservation behave as short-range correlations, they can be removed as efficiently as these, to leave a clean collective flow signal.

Then, some bad news. Since the M-particle correlation from momentum conservation is of the same order as that from short-range interactions, as for instance quantum correlations, it may bias measurements of these other correlations. It is thus worth checking that momentum conservation does not contribute significantly to the correlations measured by particle interferometry and attributed to quantum (anti)symmetrization of the wave-function. This is especially true when studying the dependence of HBT parameters on the average momentum of the particles since the correlation from momentum conservation increases with momentum [see (11) and (12)].

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